

[SQUEAKING]

[RUSTLING]

[CLICKING]

**MICHAEL** Let's get started. We're moving on. This is week four, so we're on Topic 4. They're not always going to line up so neatly with a week, but as long as we can do it, it just makes the mind a little better. And week four is about equality, equivalence and intervals. So first things first, any questions on problem set three, those who have started working on it? Yeah, Jake.

**AUDIENCE:** One of the scores we're going to do, 10.7, just behaved weird when I was testing.

**MICHAEL** Yeah, anyone else see anything weird about 10.7? Did you show the score?

**CUTHBERT:**

**AUDIENCE:** Yeah, I showed the score and I counted the number of notes, and then I actually used--

**MICHAEL** Oh, maybe I'm not thinking of 10.7 then if it didn't immediately--

**CUTHBERT:**

**AUDIENCE:** I got like three times as many notes when I did score analysis, but it didn't happen on any of the pieces, but I actually physically counted them on my hand.

**MICHAEL** Interesting. I'll have to look at that one. Yeah.

**CUTHBERT:**

**AUDIENCE:** I doesn't show the entire score.

**MICHAEL** Yeah. so it doesn't show the entire score in Jupyter. That's a feature. Thank you for reminding me. That's a feature in Version 9, which was supposed to be released before the beginning of class, but we found bugs in that where you'll be able to look at the other parts. Yeah, 10.9 is one of the longer scores. Thank you, Thank you. One of the other scores has something a little bit different, too, about it that's worth looking at. Cool. Yeah, so especially if it doesn't seem to conclude with a nice authentic cadence or something like that, then probably not.

And actually, for anybody nice, I don't have any prizes. I didn't bring any candy or anything. 66.6, the one we're working on, does it end with an authentic cadence or something else? That'll be an interesting point when we get around to automatic Roman numeral analysis, which we will get to later. The preview, I think that's piece set seven. It might be piece set eight. That's the other one, and I'll give you some notice in advance. That's the other piece set that takes a little bit more time, though, that one, you have a group for that. Great.

I can see most people are doing a very good job of keeping up with the videos and things. This weekend, by the way, you won't have any. You all need a break from that, definitely. But most people keeping up with videos. Anything about the sound to score, the thing I had hoped to get into class, but that we instead watched me, not blabber live, but blabber on video? Anything about that? There were some really cool other examples. Anybody want to share theirs on there? That one I did give sometimes just one to two words feedback.

But what were some of the things that you could use? What are some of the steps? Do you remember in going from an audio file, audio to a score? What were some of the things you needed to do? Anyone remember?

**AUDIENCE:** Frequency analysis?

**MICHAEL CUTHBERT:** What's that? Frequency analysis. Good, good. More toward the end or toward the beginning?

**AUDIENCE:** Toward the beginning.

**MICHAEL CUTHBERT:** Toward the beginning. So somewhere over here with frequency analysis. Yeah. Pitch connections. What did we mean by that?

**AUDIENCE:** I guess that's technically analyzing the notes or what notes are being produced in frequency.

**MICHAEL CUTHBERT:** What notes are being produced in the frequencies. Yeah, what's slightly different about frequency? I think you started to say, oh, that's actually the-- and then you're like, no, it's not actually the same. What's the difference between frequency analysis and pitches, musical pitches? And what's the thing we need to do, Misha?

**AUDIENCE:** I guess, some pitches or notes, they have multiple frequencies, like overtones. So we're trying to analyze that and create an instrument loop.

**MICHAEL CUTHBERT:** Yeah, yeah, when you play a really low note.

[LOW NOTE]

You can hear some of them, especially if I take one of the notes.

[LOW NOTE]

And you--

[LOW NOTE]

Get rid of the really low frequencies, you can--

[LOW NOTE]

Start hearing the thing. If you ever are writing a piano piece and you need some bells in the background, that's always a good trick. Not too many composers writing for piano and wanting bells. So good. Yeah, so we're taking out the harmonic spectrum and we're getting the pitches and things from that. What's the byproduct if we're thinking of nuclear fission? We have all these energies and we have this radioactive byproduct, what did we learn when we strip out the overtones? What might we be able to learn and get down to the notes themselves, if you do that with a whole bunch of notes or a whole bunch of sets of frequencies?

I think it was already said, but it wasn't super explicit. The word already appeared in what you said. Anyone, take a stab. What are we talking about when we're talking about overtones and harmonics? Are they always the same? Yeah, Vanessa.

**AUDIENCE:** Are you talking about, sometimes you don't necessarily want to consider overtones when you're considering the individual pitch, because they're kind of present for all of the different individual pitches that you'd be playing. To So you could when that's something that you should be paying attention to, as opposed to if it's in a chord or not.

**MICHAEL CUTHBERT:** Yeah, super. So chords definitely an issue, problem. We'll leave that off. If I can just follow up, does every D that you analyze, let's say, D octave four, does it always have the same overtone spectrum?

**AUDIENCE:** No.

**MICHAEL CUTHBERT:** What determines the difference? It's OK. Great, John.

**AUDIENCE:** The instrument.

**MICHAEL CUTHBERT:** The instrument. Yeah, so I said this was there. So one of the things, while we're going from frequency-- I should have had something up here while we're going from frequency to pitches, the byproduct is determining what instrument is being played. So we have that. Good. What are some other parts of the process that we remember? Hannah.

**AUDIENCE:** The duration and tempo.

**MICHAEL CUTHBERT:** The duration and tempo. Great. Does duration tell you the tempo, or does tempo tell you-- who votes duration tells you the tempo? Who votes tempo tells you the duration? Who votes, something else, something more complex? Yeah, you could also have raised your hand on both of them. There's kind of that weird feedback loop that you can't tell the tempo unless you have a whole bunch of durations, but then the tempo will also tell you something about, is that a quarter note. So you have second duration. Good. Any other parts that stood out? What's somebody else's?

People gave great, great examples of something that you could-- once you think you have a score, once you're at some part, you could use something from music theory or history to bring back. I'm going to put you on the spot because I remember yours was very good.

**AUDIENCE:** I don't remember what--

**MICHAEL CUTHBERT:** OK, good. Y'all do your work early in the weekend, and it's out of mind. Anyone want to make up something? I think yours was on articulations, was it not? Spiccato? Yeah, that you could find that there's-- go ahead.

**AUDIENCE:** Oh, no, I was just going to say that sometimes you might hear it as a lot of notes that can be followed by short rests and notated that way. But then based on the piece or style, you might go back and mark it as spiccato.

**MICHAEL CUTHBERT:** Yeah, based on having a whole bunch of notes that always have this large amount of rests, you might say it's spiccato, or staccato, or something where the notes are all running into each other. You might say this is legato. You might get out slurs, things. Good. Anybody else either remember theirs or something else come to mind? I'm trying to remember. There were at least three that I've never seen, that I never thought of that I was like, wow, and then about seven more that I'm like, we're on the same wavelength with each other. OK, I'll come back to that later.

One of the things you'll find, the end of problem set three asks you to write your own question. It asks you, hey, if you had a whole bunch of chorales and based on the tools you have for statistical analysis, what kinds of questions you could ask. And it doesn't ask you to answer that, but you're going to find that as the semester goes on, those become bigger and bigger parts of the problem sets, taking more time. You'll be asked to pose your own question and answer them, and both the quality of the answer, but also, the interest of the question becomes really important.

And why is that important? What are we going to do at the end of the semester? We're going to pose our own question and try to solve it. And what do researchers do in the world for the most part? They pose an interesting question, and they try to solve it. So we're going to try to make it so that by the end of the semester, you're not just students who are given an assignment to do, but also posing your own questions. And so you'll probably get sick of the fact that I'm asking you to document, write docstrings, and all that stuff in your problem sets now.

And at first, it's pretty silly because I'm asking you specifically to do one task. But then as it goes on, the more and more complex problems, you're going to be making your own subproblems. P set four is the first one where I suggest that you might write some helper functions that won't be graded. It'll be the first time where I think you probably shouldn't try to solve everything right in the answer. And so that's a good time to practice doing that type of thing.

Anything else before moving on to the third little topic from the weekend discussion, musicxml? Any thoughts? Anybody watch the interview with Michael Good and take away anything you want to share with us? He's going to drop into this class in a couple weeks, I think. He's visiting campus. Yeah, go ahead.

**AUDIENCE:** I had one thing that I kind of took away from the video. It was just how much effort goes into creating something and then trying to-- in order to ensure its like survivability into the future, how much of your own effort has to go into advocating for something when there is nothing to show.

**MICHAEL CUTHBERT:** Yeah, yeah. So he did this huge-- he said, well, I came up with this format, and then I had to market it. And I don't know if everybody knows Finale and Sibelius. Some of you all know MuseScore, and Noteflight, and things. Flat might be the things now as much. But yeah, but those were the two big leaders, and he just felt like if he couldn't get one of them to use his product, he might as well not have made it. And then just going around trying to make more, and more, and more things out of it, and it's been a super successful legacy.

Michael Good finally retired from running the a musicxml spec from taking charge, and making sure it was all working. After doing that for-- what was it? 24 years of doing it. But he finally found some poor schmuck to take on that role and agree to do it for the next couple of years. He also mentions in there there's something called MNX, which is interesting that it's really beginning to pick up some steam.

And so what musicxml was primarily about, if you recall, it was about, you have data in one application and you want to get data in some other application, and you don't want to convert it down to MIDI and loose spelling and lose all the things about tempo estimation, and beat, and staccatos, and all these other things. You want to transfer over into musicxml, was really great, is really great for transferring documents. But once you get it into your application, almost every application you use has some of internal memory representation.

You've coded things that you store. I'm going to store the name here as a string. And somebody else's, well, I'm storing it as an array of characters so I can manipulate it. Whatever it is, everybody has a different representation. And musicxml was not good for that. That wasn't what it was designed for. But now more and more people want that, and they want something. And so MNX, something like music notation next or something. The X doesn't really mean anything. The next format was coming out, and the encoding format, the final decision on that it's going to be entirely JSON based.

Who has heard of JSON? Who has not? Yeah, so it's the data format, interchange format originally for JavaScript. That's what the J, JS stands for, but now used by in almost every language. It's where XML was 20 years ago, that this is the thing that everybody knows. That's the thing now. And so trying to figure out exactly how it's going to work. But this is pretty exciting things that the next 20, 25 years, 30 years of music notation was finally decided yesterday.

So that's pretty good. It's an exciting, exciting world. And if you get very into notation and representation and things, you can jump onto the GitHub repository and start saying, no, if you represent accordion symbols in this way, it will never be useful for what I like to do with accordion. It doesn't have to be that esoteric. It could even be on octaves. The question is, we all like writing things like pitch C4, something like that. But, boy, should it be, like, name C and octave four? And therefore, you have one less thing that you have to convert from a string to an int there. And so that your program won't crash on octave negative two or whatever happens, octave 10, all the things that we always forget to do.

So anyhow, that's what's been happening a little bit in the world. OK, let's get to the main topics today. Let's start with the concept of equality. What does it mean for something in music to be equal to something else? What does it mean? What does it mean for two notes to be equal? We're a little bit less-- I don't know-- vigorous today than we have in the past, so I'm going to call on some people, just because there's not really a right, wrong answer on this. The first person I'm going to call on is Jason. Then I'm going to call on-- then I'll call on Hannah, then Paul. So what would you say means for two notes to be equal?

**AUDIENCE:** I guess abstractly, maybe just that the pitches are the same and the durations are the same.

**MICHAEL  
CUTHBERT:** OK, so pitches are the same.

**AUDIENCE:** At least for single notes.

**MICHAEL  
CUTHBERT:** Pitch, same. Duration, the same. What do we mean by pitches, the same?

**AUDIENCE:** Abstractly, mostly, if you really want to compare the frequencies, like the overtones and stuff, that might cause some issues. So I guess abstractly, just maybe going down to MIDI is a little too simple, but just name and accidentals.

**MICHAEL  
CUTHBERT:** OK, great. The name and accidental are the same. This is a very good definition. So we could completely go with that. And do you want to change anything, go with it?

**AUDIENCE:** I was going to say something really similar. Probably also consider the octave.

**MICHAEL** So octave has to be the same. Anybody else want to jump in? Do you want to?

**CUTHBERT:**

**AUDIENCE:** I wouldn't say that the accidentals have to be the same.

**MICHAEL** You wouldn't say that the accidentals have to be the same. So give me an example of something that would be the same without.

**AUDIENCE:** D-flat

**MICHAEL** D-flat and C-sharp. Great. So in this case, not only is the accidental different. What else is different? The name.

**CUTHBERT:** Good. So what can we put instead for that? What's a representation that allows these to be equal?

**AUDIENCE:** The request in the [INAUDIBLE].

**MICHAEL** The pitch class is what you might be thinking of. Yeah, pitch class, or pitch class plus octave. What's another thing that encodes pitch class plus octave? I've heard this four letter acronym used a couple of times. John?

**AUDIENCE:** I was going to say space, but it's not four letters.

**MICHAEL** It's not four letters. Space is good. Yeah, Adam.

**CUTHBERT:**

**AUDIENCE:** MIDI.

**MICHAEL** MIDI. So it's MIDI number. Good. So we could do this. So when you turn in your homework for your tonal harmony class, and it says F major, 5 to 1, write this, and you put in-- and you get it wrong, marked wrong by your teacher that wasn't 5 to 1, I think I got-- no. Well, that wouldn't be 5 to 1 anyhow. That's what I'm looking for. Do all of these notes have the same MIDI number as this? Yeah. Good.

So maybe we don't want to go with that, because there's a place where your teacher says it's wrong. On the other hand, what are the consequences of that, of not saying that MIDI? Are these two notes the same?

[MUSICAL NOTES]

Of course, when I do this, it's always a trick question. What do you think the answer should be? Yes, but we already said we're not going to count that the same. And I played a C and a B sharp. You all you all knew that, right? I conceptually flipped the n harmonic between the two. So we've worked ourselves into a real bind. I think this is a useful exercise, but the answer really comes with, we don't really, really know. And we can define different cases where we use the words-- we don't use the word equal too much, but let's say the same, the same. Yeah, go ahead.

**AUDIENCE:** I had a good question about what cases would the details of the meaning of musical equality really matter?

**MICHAEL** Great. We're about we're about to get into that. What cases? Let's we'll open that up as a topic to think about there. I have some in my head, and it's going to be-- you're one slide ahead of me. But maybe everybody sing this with me.

[MUSICAL NOTES]

[SINGING]

Well, the man sang with me, but the woman didn't, unless we have a saying, "sing this" doesn't include the octave. But also, we can think of a lot of different things that might be slightly different with each other. We can also say that here are two notes. Bom, bom. Are they the same? Yeah, in some ways, but also, no. The first one was on beat one. The second one was on beat two. There might be cases where we want to not just consider the duration, which we didn't even talk about, because I already began this semester with the trick question of how many notes. Do we remember that?

Good, good. I'm starting to see smiles and things and not just scared, he's going to call me. But yeah. But not only duration, but we might also consider what beat it's on, that we might not say that to be the same. So there might be different cases like that. Here's an example from one of these. Oops. Well, it's even more complex than I expected it to be without that. Great. Who is my designated "tell professor when he's projecting without projecting"? That's all of you, right? You'll all you'll all take it on yourselves to say and save time. Good.

So this is one of these weird experimental pieces from the '60s and '70s and things called Burdocks by a composer who's a Dartmouth professor of philosophy, actually, Christian Wolff, a friend of John Cage's. And this is typical things. It's in a whole bunch of movements. But very famously, for me, is the opposite of equal, is the instructions for number three. Each player is to make about 400, 511 sounds. Each sound different in some way.

And I remember talking with Professor Wolff. He was a guest speaker in one of my composition classes once, and he was talking about the interpretation that this movement ended up being one of his favorite ones, watching it performed to see what people did. And in one case, a player had a bucket, and then had another bucket filled with water, and took an eyedropper, and dropped one drip of water into the other bucket, and hit it, dropped one, hit it, dropped one, because the frequency would change. And after 511, it changed a lot.

And so he thought-- the composer was saying, oh, I thought this was very interesting. And other people did all kinds of things. But as you can imagine, you get your piece played with a larger ensemble. And some people are really, really into this kind of music. I can find it. If you've never performed a piece like this, don't knock it till you try it. It always sounds kind of, I won't enjoy it. Then you go and do it, and you're with all your friends, and you're trying to follow these instructions, and it can be quite wonderful.

But there's always somebody who's not going to want to do it. And they said there was-- I think it was a trombone player who just sat there, bap, bap, bap, bap. And the composer only came to the performance, not any of the rehearsals. And he said he got so furious at first, because the only thing, the only constraint he put on him, on the performer, was that each note had to be different in some other way. And so the only thing, you can do anything you want, choosing not to.

Then he said, but of course, it's 511 notes, so I had a long time to think about it. And after a while, I began to really, really like what he was doing, because every one of them was different. Because first, it was a bap out of the blue. Then it was a bap that had been preceded by a bap. Then it was a bap that had been preceded by two other ones. And the one that you hear in the 100th time is very different. So this is all to say that we can think of equality in many, many different ways, and that there could be different cases where you want to do something different than the other.

Let's go ahead and start a Jupyter Notebook. You guys seem to be pretty fast at getting this. And we want to figure out some definitions of equal that Music21 has used, but just only to think of what kinds of assumptions were built into the system, even though they're not-- you could have just about any one. So I'm over here. And just to review, how do we test in Python that x equals 5? What's that? Just shout it.

**AUDIENCE:** Assert.

**MICHAEL**  
**CUTHBERT:** Assert x, 5. No. If you were doing in an if statement, what would you do? If?

**AUDIENCE:** x equals, equals 5.

**MICHAEL**  
**CUTHBERT:** x equals, equals 5. Good, good. Yeah, problem is I have to remember where my trick questions are and where we're going to the real ones so you can do it. Good. And Python's a little bit interesting that we can create two mutable arrays. We'll say A is 3, comma 4 B is 3, comma 4. Does A equal B?

**AUDIENCE:** Yes.

**MICHAEL**  
**CUTHBERT:** You say yes, no. We can hit Shift Enter and get it. In this case, yes. I think I can do this up here without anything going. Can I make this any bigger? Yep, I can. Let's do this in JavaScript. Let x equals 3, comma 4. Let y, B equals 3, comma 4. A equals, equals B, false. So two very well often used languages, both kind of scripting type languages. This is the first time I've shown my console in a while. Yeah, that end up with very different results on that. Let me just make a note for myself to make sure to do that. Yeah.

This is, by the way, one of the first mistakes I always make when switching over to JavaScript. Now, Python has a second equality operator. So there's equals, equals, and there's is. Yeah, so A is B, false. So x is 5, yes or no? You get all kinds of warnings now if you're on a new Python saying, that's not really a great way to do it, that's not really a great way to do it. Hey, stop doing this, Michael. Stop doing it. Oh, but yeah, it's true. So various things, but if we-- so x equals 5, y equals 5, x is y. True, actually. It's very odd. There's all these weird things that happen.

So you'll find that there are other things that come up quite often that are all the choice that somebody made at some point to do it. So let's do music equivalence ops from Music21 import note. Is that big enough? No, it got smaller. And here's something that I am going to do when I'm in class. N equals note dot note. Capital N equals note dot note. And notice I'm not calling it. What I'm saying is the variable capital N can now be used as the class note dot note, so I can do things like N, C4, plot show, and same as anything else. I probably shouldn't have called it that, and works the same.

So give everybody a second to catch up. I'm going to create two of those C4s. Going to call it C. I'll call it C2. And what do you hope that the answer on this is? True. By the way, I am running Music21 version 9, and you all are running version 8.1 or 8.3, or something like that. We changed a few equals things, so if something up here is different from yours, shout out, and I'll try to remember if that's the place we change it. But I don't think we'll get too much into that.



Good. So two notes that are the same are there. Let's call C3 equals N in the same note an octave higher. So go and type that. Well, we hope. If you're ever behind, so false. They're not the same operation. Does the duration need to be the same? So let's just remember that C is the same as C2. Let's say C2 duration quarter length equals 4.0. Anyone remember what's the default quarter length in Music21?

**AUDIENCE:** One.

**MICHAEL**  
**CUTHBERT:** One. That's why it shows a quarter note. So we'll make it 4, which is what?

**AUDIENCE:** Whole.

**MICHAEL**  
**CUTHBERT:** Whole note. Good. C is the same as C2, false. So good. We so a lot of us have the same sort of intuition as each other. Maybe that's bad. Maybe that's groupthink. So does anything make an object the same? We'll say C4 equals-- we'll create another. In this case, C4 is C4. C equals C4. What about C? Here's something you probably haven't seen. Lyric equals hello. Hopefully, that should work. And C4's lyric, I don't know. What's your favorite language? French will do. Bonjour. Does C equals C4 now? Is that the same on yours?

So we've decided that we the Music21 community-- I can't remember if I decided this one or not, but I'm living with it-- that the lyric is not an essential part of the quality of a note. The two notes can have different lyrics, but still be the same thing. Weird Al Yankovic has been working on that for a very long time. You can do the same song, different lyrics, still the same song. OK, well, let's do this one. bs, not for B sharp. Let's create a sharp.

And oh, if I want the same note, let's see how anal your music theory teaching was. If I want the same note as C-sharp 4-- sorry-- C4 as a B-sharp, what's the octave?

**AUDIENCE:** Three.

**AUDIENCE:** Four.

**MICHAEL**  
**CUTHBERT:** Four, three. Well, first, let's create B-sharp 4. And by the way, this is one of these things. I think that there's a lot of consensus on this, but not unanimity. So somebody might have taught you something else. But the octave number refers to the letter name. And so when you put lots of sharps on it, even though this is the same note as C--

**AUDIENCE:** Five.

**MICHAEL**  
**CUTHBERT:** Five, it's still in octave four because the B is an octave four. These are all weird things. So we want bs-- well, we can say dot pitch dot octave equals 3. Now, bs should be the same note. Ooh, I just slipped. It should be the same note as C-sharp. So if it's the same note, or C natural. So it's the same note, then these two should be equal. And they are not. So how do you work in a world where you want this to be equal?

One way you can do this, a nice thing about a lot of languages like Python is you can do something called monkey patching. So we can just create a new equality thing, return whether or not dot pitch dot MIDI equals B dot pitch dot MIDI. Whose idea? And then we can say that n, which is the same as note dot notes, it's special thing that you run, underscore, underscore eq, which is run anytime you check equality, equals new eq. So now, anytime there's want to do equality, we can check that.

So this is size I use German when I need to do things. E-sharp equals that, and F is our normal F. Now we can see is E-sharp the same as F? Yes. Just before I do it, you will create things like this all the time to deal with your definition of equality. Do this much less often, because I'm not even sure if note dot show will work anymore, or if there's other things there. Yeah.

**AUDIENCE:** What did you say about the B-sharp 4 going back to C4?

**MICHAEL CUTHBERT:** So this definition of octave, which I used quite a bit, has the notion that the octave defines a range of staff spaces. So if we're in treble clef, this is all octave four. October 4th. It doesn't matter how many flats or sharps you put down anything. Anything that's written on those staff spaces in treble clef will be octave four.

**AUDIENCE:** So in this case, writing B-sharp 4 would be pointing outside of the staff.

**MICHAEL CUTHBERT:** No, so B, you first see, OK, I want octave four, and then I want the B in octave four, so it's going to be this one. So B sharp would be this one, even though the C in octave four is down here.

**AUDIENCE:** But if you play the B-sharp 4 in this, it would still--

**MICHAEL CUTHBERT:** Sound the same as--

**AUDIENCE:** A C5.

**MICHAEL CUTHBERT:** A C5. Yep, great. Like I said, you could probably find somewhere someone who's not using that definition. And you can definitely find a lot of places where this has not been well defined in lot of textbooks, but yep. Any other questions on that? We can try. Certain things are meter time signature. Anyone play this? Is 3/4 the same as meter-- this is why I abbreviate these things-- time signature 6/8? Copy all that. Oops. Yes, it's exactly the same if you don't import meter. They're both an error from Music21. Import meter.

Yeah, so it's not just based on the duration of the meter. You can try to figure out, there's some ways that you can define time signatures such that they might look the same but do different. Is 2/8 plus 3/8 the same as 3/8 plus 2/8 and so on. Any other questions on-- oh, and even in this case, by the way, what we've redefined to be MIDI is here. I'm going to put this up here so it's next to it. eis is f, false. Yep.

**AUDIENCE:** I actually did 3/4 is equal to 3/4, and it returns false. So that's what's happening?

**MICHAEL CUTHBERT:** I think that might be one of the places that was fixed in version 8, in version 9. Yes, so that was changed in version 9. I think that I remember that was some time over the holidays. I got an urgent call from somebody who was like, I needed this to work, and it didn't. So that's good. There's a lot of things that are still up in the air, and there are all things that people can debate. So equal's kind of slightly arbitrary in this case.

But let's do one exercise. It'll take like five minutes to do. Either get a blank document. You can probably just jot it down, also, in the Jupiter, or it's more fun with a blank sheet of paper or something. And you're going to hear a chacoan, which is a theme that repeats over and over. And what I want you to do is, there'll be some letters. And maybe don't try to count the number of measures. Just say just before A or a little bit after A or B, these rehearsal letters. And I want to find out where you hear the chacoan theme each time. It begins at the beginning, so you all get that as a freebie. But I want to try to do it. And then if it's the chacoan theme but transformed, what's transformed about it?

Anybody know that piece before? We played it. A few people? Anyone playing it right now? I believe it's on the wind ensembles next concert. I saw that around. Great. So I feel like it's a great piece, one of Holst's greatest, maybe after the planets and things, but one of his wonderful pieces. So about how many did people find? Tell me 2.2.

**AUDIENCE:** Seven or so.

**MICHAEL  
CUTHBERT:** Seven or so, about?

**AUDIENCE:** I was going to say, it was pretty much there the whole time.

**MICHAEL  
CUTHBERT:** It's there the whole time. Good. Anyone find a place where it wasn't?

[MUSIC PLAYING]

What's different on the second entrance?

[MUSIC PLAYING]

John, go ahead.

**AUDIENCE:** They harmonize the main thing.

**MICHAEL  
CUTHBERT:** So it's been harmonized. So something can be-- a theme can be the same theme, but harmonized. What else is different? Anybody?

[MUSIC PLAYING]

Yeah.

**AUDIENCE:** It's the instrument that plays it.

**MICHAEL  
CUTHBERT:** It's what?

**AUDIENCE:** It's the instrument that plays it.

**MICHAEL  
CUTHBERT:** The instrument that plays it. Good.

[MUSIC PLAYING]

I always think that when the clarinet is going to enter it, but then somebody, the other one does. What else has changed? Do you remember where it started? Yeah.

**AUDIENCE:** The dynamics.

**MICHAEL** The dynamics. Good. So dynamics we consider something sometimes to be the same, sometimes not if the  
**CUTHBERT:** dynamics are different.

[MUSIC PLAYING]

Now we're back to a very low octave, which we weren't in every time.

[MUSIC PLAYING]

What's different this time?

**AUDIENCE:** Duration

**MICHAEL** Duration. Or we could say spacing between the two, because the duration hasn't really changed. So that's  
**CUTHBERT:** another thing that we might think of as the same but different.

[MUSIC PLAYING]

Register instrument again.

[MUSIC PLAYING]

So I'm going to fade it out for a little bit so that we can just talk. So all of these things that we wouldn't say it's playing the exact same music. But we're saying in some sense, they're playing the same thing as before. And we'll get one more coming up in just a little bit.

[MUSIC PLAYING]

This is very beautiful, anyhow.

[MUSIC PLAYING]

What's here?

[MUSIC PLAYING]

Does anyone hear the theme now?

[MUSIC PLAYING]

Nobody?

[MUSIC PLAYING]

Holst seems to think that the theme is still playing here. Says he doesn't-- yeah, Sean.

**AUDIENCE:** It's like the concept of the theme is still there. It's just kind of a bit more of a major ish.

**MICHAEL**  
**CUTHBERT:** A little bit more major ish theme? Yep. Good. Yeah.

**AUDIENCE:** It's inverted.

**MICHAEL**  
**CUTHBERT:** Is it inverted?

[SINGING]

What do we have here? Instead of--

[SINGING]

So it's an inversion. Is playing something upside down the same as playing it?

**AUDIENCE:** Depends.

**MICHAEL**  
**CUTHBERT:** Depends. It really, really depends. If I say, let's all sing happy birthday-- let's see if I can do this-- and you all go happy birthday to-- how's it go? To you, you'd be like, OK, you just ruined my birthday. Right? Right? So it's not always the same. Yet there are some cases where composers think of it as the same. By the way, if you don't have that, one of the things nobody said was it's the same when the rhythm is the same, if nothing else.

[SINGING]

So it is even still the same rhythm, even if it wasn't fully inverted. But yeah. So things that are sometimes we want to consider the same, and sometimes we don't, we don't tend to call them equal very much. We tend to say that they're equivalent, and they're equivalent in some way. Or what we'll often use-- I'm putting this up so somebody might put this back down-- is that they're equivalent under something. Let me take something that you might know right now, just looking up there. It's a very musical-- we just did a follow of John Cage, and the time right now is that.

Anybody prefer that time? What's the equivalence here? How is it that some people might say, especially if I put that on, that these two are equivalent? But let's get rid of that for now. Yeah. Go ahead.

**AUDIENCE:** One's using a 12 hour time signature, and one's using a 24 hour time signature.

**MICHAEL**  
**CUTHBERT:** Yeah. So 12 hour can-- each time in a 12 hour in a-- we could call it an AM/PM world-- has an equivalent in a 24 hour-- you could call it military time, whatever you want to call it-- world, so that there is an equivalence that allows the mapping back and forth between them. Not all equivalences allow a perfect mapping. Good. Mathematically, what do you need to do if you're at 2:05 PM? What do you do to convert that to a 24 hour time?

**AUDIENCE:** At 12:00.

**MICHAEL CUTHBERT:** At 12:00. Good. And what do you do if you are at 14:05 PM and you need to do it? You can subtract 12, or you can think of it as mod. Yeah, is it mod 12 or mod 24?

**AUDIENCE:** Both happen to work.

**MICHAEL CUTHBERT:** Both happen to work. Do they? No, mod 24 doesn't work because 23:50.

**AUDIENCE:** OK, yeah, but in that case, it works.

**MICHAEL CUTHBERT:** In this case, yes. In this case, it works. Yes. Good. Always solve for the most specific case you can. No, of course, we'll know. We'll try to solve for the most general case. So mod 12, there's some other little thing that you have to do on this if you're at 12:05 PM. Let's see. Yeah, in that case, no. I can't remember. Anyhow, there's something off by one error, so don't follow. Don't try to figure them off at the-- there's a particular case where if it equals 0, you're going to want to add 12. So it's mod 12 plus-- I can't remember. You'll figure it out. I know how to do it in music and stuff.

So let's think of some musical equivalence classes. One of the ones that we've put up here, and so one of the reasons we debate a lot about equals is that equals is trying to create a set of equivalence classes once and for all. This is better. I'll keep this one in the center. So here's a particular thing that we can debate whether or not they're equal. Music21 has decided that they are not, but that could be a debatable. But I don't think it's debatable that there is some form of equivalence between these two. What do we want to call that?

**AUDIENCE:** Enharmonic?

**MICHAEL CUTHBERT:** Enharmonic equivalents. Good. Enharmonic equivalents. They are enharmonically equivalent. Somebody told me if I got my Es and As right. Yeah, I think this time. We can also sometimes say they're equivalent under spelling, under lack of spelling or something equivalent with spelling. Good. What's another equivalence class that-- we're going to go really fast, so let's start thinking. What's something else that we might sometimes think is equivalent? We said inversion. Sometimes, but often not. Good. What's another?

**AUDIENCE:** Duration?

**MICHAEL CUTHBERT:** Duration. Sometimes we don't care how long two things are. Good. John?

**AUDIENCE:** Key.

**MICHAEL CUTHBERT:** Key. Ooh, that'll be good. Great. I heard another one. Just shout.

**AUDIENCE:** Octave.

**MICHAEL CUTHBERT:** Octave. I should have chosen the one without staff lines. Twice as much space. Keep going, everybody. What's that?

**AUDIENCE:** Instrument?

**MICHAEL** Instrument. Yeah. Sometimes it doesn't matter what instrument is playing. We'd say it's the same thing. And  
**CUTHBERT:** sometimes when you hire a musician to perform at your wedding, and they show up with an accordion and a bagpipe, it's not equivalent. Right? Good. What other things that we might say?

**AUDIENCE:** Context.

**MICHAEL** What's that?  
**CUTHBERT:**

**AUDIENCE:** Context.

**MICHAEL** Context. No, too broad. Too broad. Good idea. Give me one context that might be--  
**CUTHBERT:**

**AUDIENCE:** Where you're coming from and where you're going to.

**MICHAEL** Where you're coming from, coming from, going to. So the same song is the same under equivalence of car stereo  
**CUTHBERT:** versus home. It's interesting that we might think. Give me a context where listening to the same song in your recording studio or at home versus your car might be a difference. The same song might not be equivalent under that.

**AUDIENCE:** If the car has police sirens in it?

**MICHAEL** If the car has police sirens in it. Good. I was thinking, if you're trying to mix your composition for submission,  
**CUTHBERT:** does it matter if you're listening at home in stereo with your very nice headphones? Is it the same piece as if you were-- let's hope you're not doing it while driving. You're in the passenger seat, but you're doing it on a noisy road. Yeah, so the amount of background noise might be a context that changes whether or not you think this piece of music is the same. Yeah.

**AUDIENCE:** I was saying if you're looking at a pitch, what the chord is coming from.

**MICHAEL** Yeah, I knew that. I knew that. I knew you meant that, but yeah. So preceding versus following. Good. Other  
**CUTHBERT:** things. This year we've been super great. Yeah.

**AUDIENCE:** Articulation.

**MICHAEL** Articulation. Good. Articulation. Good. Shout some more.  
**CUTHBERT:**

**AUDIENCE:** Tempo.

**MICHAEL** Tempo. The same piece is the same piece if you rush through it, sometimes. Sometimes not, but yeah, good.  
**CUTHBERT:** Anything else? You can keep thinking of some more. I want to use them all up, because I think we're going to be asked to come up with at least one. That's great. So this is really, really good. You're going to be reading. Aha! You all got to be faster. You're all going to be reading another section from this book by Dmitri Tymoczko, a geometry of music, and a second part of Chapter 2, which you read earlier when we were talking about those flipped inversions with the mirrors.

And he defines five points of inversion, five equivalence classes that we think of quite often in music. And he gave them. The ordering isn't the greatest ordering in the world, except it makes it very easy to remember. OPTIC. And I'm going to add a sixth one because I think it's very important to. The octave. Quite often we don't care. We consider the two musical elements, two chords to be the same if they're in different octaves. Or two notes, the same note or equivalent note. Permutation. What do you think that means? Yeah, Adam.

**AUDIENCE:** You have a chord and some inversion of it. Perhaps you want to consider them the same.

**MICHAEL CUTHBERT:** Yeah, so we have an inversion of a chord. You might consider it the same. The order that goes only-- that would only work if you also have octave.

[MUSICAL NOTES]

Is there some way that those are the same? If I told you the first note was C, they're both arpeggiated C major triads. You can hear that. Does it matter? Sometimes it matters if, well, you're arpeggiating it up, and sometimes it matters if you're arpeggiating it down. If you're doing your theory analysis, does that matter if you're trying to hit a chord note? No. So sometimes, on the other hand, let's stick with happy birthday, I'm going to say happy birthday to you. But I've run it through my Python scramble thing. I'm going to do every note in a different order.

We would not apply that equivalence class at that place, so the permutation is something that, quite often, when we're just analyzing the chordal structure of a measure, we don't really care what order the notes come in. We just want to see what are all the notes in the measure, and then figure out what chord it might be. So the permutation. Transposition. Who shouted key? Yeah, John. Were you thinking of the same piece in two different keys? Could be?

**AUDIENCE:** Yeah, that and saying, it's kind of synonymous to enharmonic, so if you're like G-flat major or F-sharp.

**MICHAEL CUTHBERT:** OK. Yep, also. Great. But sometimes it doesn't matter actually what key you're in. If you're a singer and you show up, and people used to be able to-- pianists accompanists, collaborators used to really know how to do this and stuff. You've been practicing all week in G major, and the singer comes to the concert and says, I woke up with a scratchy throat. Can we take it down to F today? And they play. And you're still playing in some ways the same piece. So the transposition might not matter in some cases. In some cases, it really matters.

One place it really matters is if you're playing the viola in the orchestra, and you choose to transpose and nobody else does. I think people will notice. So transposition, again, these are all equivalence classes. I didn't do a good job of this in other years of lecturing. So I want to say, now, these are things that in certain contexts make sense to apply, and in certain contexts, do not make sense to apply. And so transposition is one here. Inversion we've already talked about. In some cases, it makes sense to apply. In some cases, it doesn't.

Want to say there are two different-- a number of different definitions of inversion. And that's actually one of the things that happens. One is something that we call a tonal inversion, where you say tonal-- I'll put it in the key signature-- something like that. Instead of going up four notes, we go down four notes, and then go down one more note. In this case, it's exactly the same. Let me go. Let me keep going up here. I have to go one more note up. Assume these go here. Eventually, you get to a point where this is a half step, and this is a-- that was good to do ledger lines-- whole step.



In a tonal inversion, you just care that you went up a step in one, and you went down a step in the other. If you went up four steps, you don't really care if it's a perfect fourth or whatever. In an atonal, or post-tonal, or non-tonal thing, we would really want to make sure that-- where did I screw up? There has to be some place, because this is an octave up. I'm pretty sure if you go up an octave, you end up going down in octaves.

**AUDIENCE:** The first series of notes in the second, the right hand one.

**MICHAEL CUTHBERT:** Sorry. Now I'm in a post-tonal world, so now I need to write all my accidentals. Oh, there it is. That's the whole step, and so that would have to be the half, whole step, whole step, and there. Yeah, so those the two kinds of inversion that he's talking about. And sometimes it makes sense. Sometimes it doesn't. If you listen to the music, the post-tonal music of the early 20th century, they use inversional symmetry a lot. And so if you're trying to figure out, oh, how does this relate to something I've heard before, almost always try, well, is it the inversion of it?

And there's some cool things that-- here's four triads. You might know them by name. You don't need to know the name, but what type of triad are they.

[MUSICAL NOTES]

Anyone, shout it out.

**AUDIENCE:** Major.

**MICHAEL CUTHBERT:** Major.

[MUSICAL NOTES]

Minor.

[MUSICAL NOTES]

**AUDIENCE:** Augmented.

**MICHAEL CUTHBERT:** Augmented.

[MUSICAL NOTES]

Diminished. So there's four different triads. Major, minor, augmented, diminished. A lot of people think that major and minor have more, something more in common than the other two, because this goes what's up a major third, up a minor third. If you start and instead go the opposite way, and you start here and you go up a minor third, and then you go up a major third, you end up with a minor triad. So the minor triad is the inversion of the major triad.

And therefore, they both have the exact same interval content. They both have a minor third, a major third, and a perfect fifth within them, whereas an augmented triad cannot be inverted to become a diminished triad, or to become a major or minor triad. So they don't have an equivalence class between them. So this comes up often.

Last cardinality, how many notes there are. Sometimes it matters. Sometimes it doesn't.

[MUSICAL NOTES]

Versus.

[MUSIC PLAYING]

In some ways the same? Yeah. Also, if you put it together with octave and cardinality-- I'm not going to run over there again. I need to put my keyboard here-- a four-note major triad, C-E-G-C, now you say that octave doesn't matter. So now it's C-E-G and a second C here. And now you really want cardinality not to matter if you want to say, is this the same as some other C major triad. And the last one I think is really important. Dmitri Tymoczko comes from a slightly different practice world. He just assumes that spelling is equivalent, or what we call MIDI equivalent, or enharmonic equivalent.

I should have called it enharmonic, but I can't figure out a way to make an E into the optic acronym. So does enharmonic equivalent, does spelling matter? And one of the things in his world, which is very different from Music21, but in problem set four, you're going to be working in the world of optic, of geometry, of music, the spelling equivalence is always assumed. Yep.

**AUDIENCE:** Poetic? P-O-E-T.

**MICHAEL**  
**CUTHBERT:** P-O. Yeah, but then I would have to rearrange his. Yeah, no, that's really, really good. There's another one that we can start putting in. We can start thinking of timbre equivalents. We often think of that, all these other things that you wrote up there. But these are the ones that will give you the most hard time on the next problem set. There's not time. So we'll end this up, end a little bit early for the first time in a while. Cool. We'll get to working with intervals next class and filtering streams. Thanks, everybody.